To provide an example and referring to *Figure 3.6*, we can see how it is possible to obtain *80* possibilities corresponding to his second line.

If a box is occupied by *3* particles out of an available *4*, the simple combinations of *4* objects with *3* by *3* (as taught by the Combinatorial Analysis) are given by the binomial coefficient

$$\binom{4}{3} = 4,$$

and the four possible groups of three numbers have five positions from which to choose. From here *4×5=**20*** possibilities for the group of three numbers.

The single remaining particle has the possibility of the four remaining locations, and therefore has *1×4=**4*** possibilities.

The product ***20×4=80*** gives us the total possibilities in the case that the particles arrange themselves in two groups, one with three and one with a single particle and having five boxes suitable.

It is easy to verify that we will obtain the same result considering first the single particle having five boxes suitable (five possibilities: *1x5=**5***) and after the three having the four remaining (one is occupied by the single particle, therefore *4x4=**16*** and ***5x16=80***).

Applying the procedure line by line it produces the results shown.

Part 4 (of 4): Chance

4.1 CHANCE

A sharp-shooter shoots at a target with an excellent rifle: he aims carefully, chooses the moment when his breathing will not interfere and the amount of force with which to pull the trigger so as not to move the barrel, fires the shot and hits the bull's-eye.

Immediately afterwards he takes all the same precautions, but the shot ends up being slightly off target: it could have been a slight disturbance to his sight, an involuntary variation in his breathing, an imperceptible abnormal movement of the finger, a very slight unpredictable wind, or who knows what else.

The causes are many and imponderable, slight if each is considered in itself, but interacting differently each time, ensuring that each shot has a different fate.

This complex of innumerable causes of disturbance which are not controllable or predictable, and which, not being able to take each into account, one by one, are called the *Law of Probability (Gauss's Law)*[10].

Probability for the reasons given, and *law* thanks to Carl Friedrich Gauss (1777-1855) who wrote an equation capable of taking into consideration, in a global manner, all those fleeting causes so as to be able to predict with near accurate approximation how the shots will arrange themselves, percentage wise, round the target with different distances from the bullseye. The approximation will be more accurate the greater the number of shots that are fired.

Let us assume that the target is as represented in *Figure 4.1* and is divided into two parts by means of the section *AB*, and that our sharpshooter fires many shots; after which we count the number of shots which hit the target in each half:

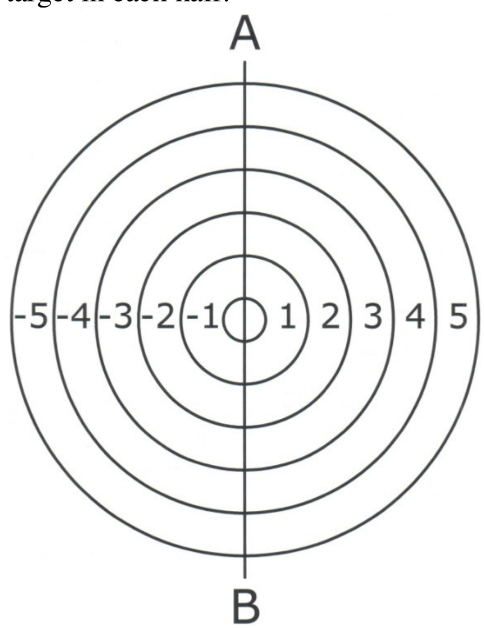

Figure 4.1- The segmented target

If the reasons for the error are truly *random* (rifle without defects, such that it does not tend to deviate the shot systematically and neither does the sharpshooter have an analogous defect; there is

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[10] *The example of the sharpshooter was published by Engineer Mario Manaira in N° 256 of "Journal of Mechanics" together with our first article concerning thermodynamics, more than half a century ago (1961)!*

not a steady wind, etc.; in other words there does not exist a cause which always influences with the same bias, *called a systematic cause*) we could note the following:

1. The shots will be greater in number in the first band round the center;
2. The shots will progressively decrease in number in the subsequent bands as these distance themselves further from the center, until there are very few in the bands furthest away;
3. The shots in the two halves, right and left, in any similar band, will tend to have the same number and will even be identical if sufficient shots are fired.

It is therefore possible to represent the phenomenon graphically, as in the following figure:

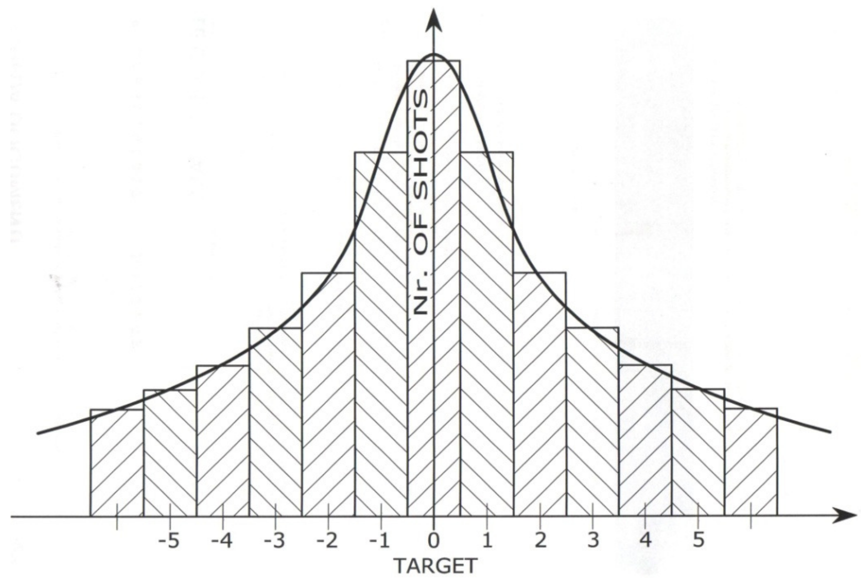

Figure 4.2 – The random distribution of the shots in each band and the Gaussian distribution that would be obtained with an infinite number of shots fired.

If the marksman were less capable, the concentration of shots near the zero on the abscissa would reduce and the curve would flatten itself, while maintaining the characteristics given and represented in *Figure 4.3*. The first observation is that the maximum height of the curve constitutes the "target", in other words the goal of the operation, while the absence of systematic causes (in antithesis of randomness) ensures the symmetry of the curve with respect to the vertical which represents our target zero.

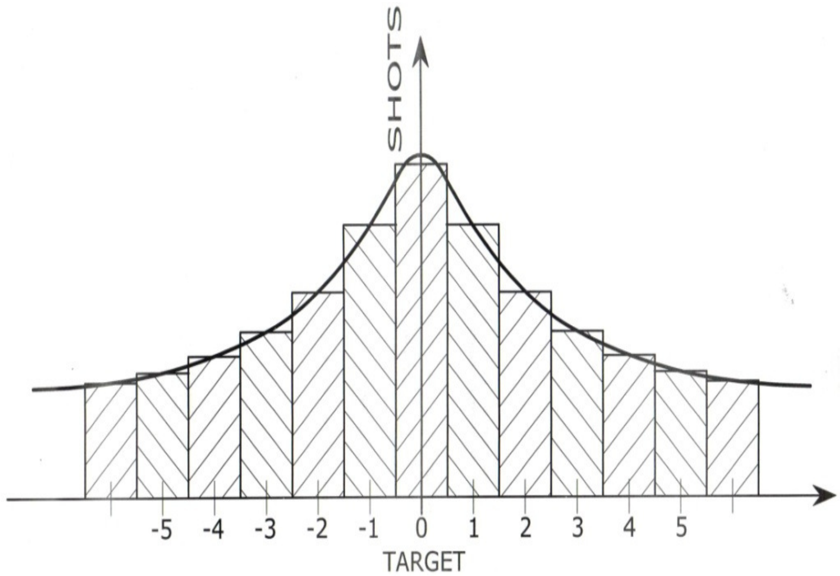

Figure 4.3 - If the marksman is less skilled, the Gaussian flattens.

In the case of a systematic cause of error, the curve loses its symmetry: if we assume that the test is performed with a constant wind from left to right, the graph will take on the shape of *Figure 4.4*:

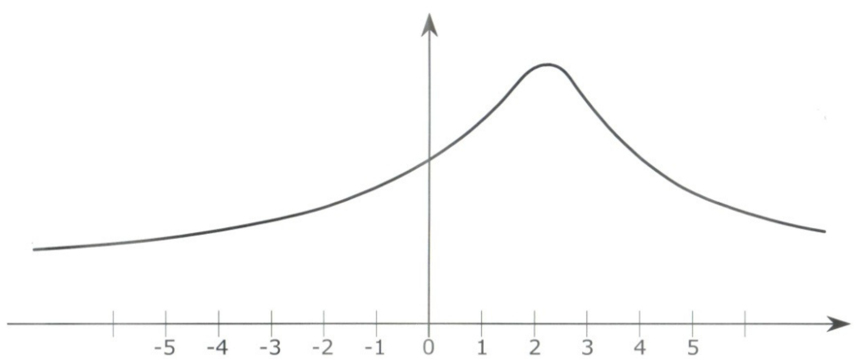

Figure 4.4 – When the Gaussian is asymmetric it implies that the phenomenon is not "entirely random"[11].

Let us suppose now that our sharpshooter is blindfolded, the target becomes very large and is moved: he will have to shoot blindly (randomly), left and right, high and low. Given that the Gauss

[11] Gauss suggests that the analytical expression of the Law of Randomness is the function.

$$y = e^{-x^2},$$

where it can be seen that the curve is symmetrical with respect to the axis *x=0,* and decreasing both towards the left and right of this line and has a maximum for *x=0*.

It can be shown, further, that the area subtended is

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi} \ .$$

To ensure that this area is equal to unity, as opposed to $\sqrt{\pi}$, appropriate steps can be taken, which, without changing the general properties illustrated, give the *normalized* Gauss's Law.

function still applies, the probability curve will flatten itself, maintaining the essential characteristics; in particular the two tails which will tend towards a tangent with the abscissa tending towards infinity, a maximum point, a point of inflection and the other characteristics illustrated in *Figure 4.5*.

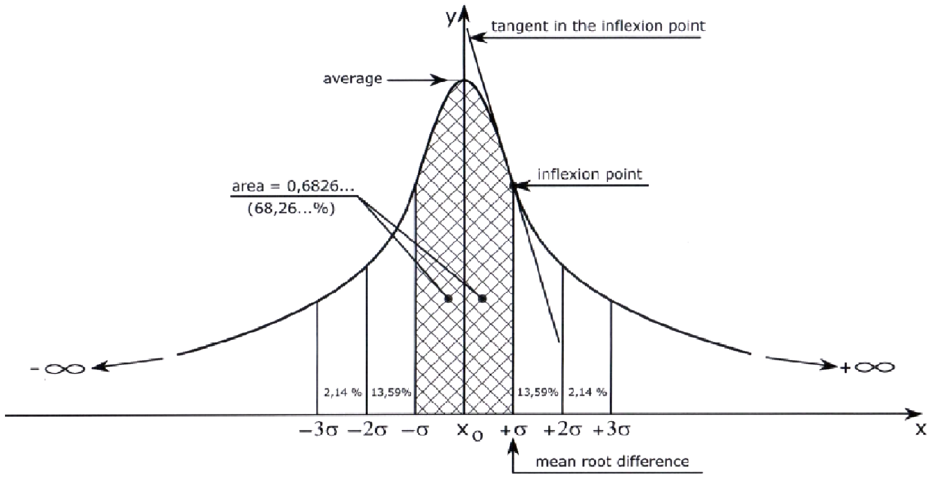

Figure 4.5 – Typical characteristics of a normalized Gaussian.

Supposing once more that the Gauss function still applies, it would be logical to expect a distribution with a curve that is so flat that it will be difficult to see a maximum point corresponding to the center of the target; it will be necessary to fire enough shots so as to occupy every position on the abscissa and to have hit with 100% certainty the bull's-eye.

This implies that *everything is possible*, ***as long as an infinite number of shots are available (using rhetorical language).***

4.2 SOME PROPERTIES OF RANDOM EVENTS

The perplexities regarding the applicability of chance, as referred to the blind sharpshooter, depend on the fact that the Gaussian assumes that ***programming*** has been applied to reach an objective, which implies that the operator ***is conscious of the objective***; an element which in this case is absent.

Both the existence of a program (the sharpshooter sets out to hit the bull's-eye) and the existence of an objective (the card with circles) appear to be essential to be able to talk about chance.

Another example: let us imagine a machine programmed to produce a certain mechanical piece: *the program* is the design of the piece written in machine language and the *objective* is the production of the piece. In mass production we will find that it is the case that, despite the work conditions being maintained the same, each piece will be different to the other, to the point that the pieces which exceed the tolerances (which would not allow them to be interchangeable) will be rejected.

Innumerable examples could be presented identifying in every case these two characteristics: *a program and an objective*.

Statistics also operate in reverse: from the measurement of a group of subjects it creates a bar chart: its envelope will be the curve of the random distribution. It will give us the average of the values measured; if the curve is symmetrical it will tell us that the phenomenon is not influenced by systematic causes; further, it will tell us the value of the standard deviation, etc.